

**Politecnico di Milano**  
Dipartimento di Scienze e Tecnologie Aerospaziali  
Prova finale: Introduzione all’Analisi di Missioni Spaziali  
Docente: Massari Mauro

Elaborato n. C13

Autori:

|  |  |
| --- | --- |
| 10694997 | Pala Silvia |
| 10711624 | Turcu Alex Cristian |
| 10730510 | Vanelli Paolo |

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# Introduction

The project aims to study, optimize and choose various orbital transfer strategies, having as initial data a point on the initial orbit, which position and velocity vectors are given, and a point on the final orbit, which is defined by its orbital parameters.

First it will be analysed a strategy based on a set of standard manoeuvres.

Several other alternative strategies have been examined to try to optimize the two most significant parameters in their distinction: the manoeuvring cost (the total speed gap required to complete all the orbital changes) and the operating time (from the start point to the final point).

All calculations and plots are made using MATLAB software.

# Initial orbit characterization

## Initial orbital parameters

The assigned starting position and velocity vectors are the following:

It is possible to calculate the orbital parameters assigned to this specific couple of vectors:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| 8369.7448 | 0.1097 | 0.8487 | 1.5339 | 1.1849 | 1.8025 |

## Data interpretation

The starting geocentric orbit is elliptical, with an eccentricity value between 0 and 1 and a specific energy of:

It belongs to Medium Earth Orbit (MEO) category, as its apogee and its perigee are inside the range of 8000 – 42000 km:

According to the given value, it is nor a polar nor a geo-synchronous orbit and has a period of:

## Graphical representation

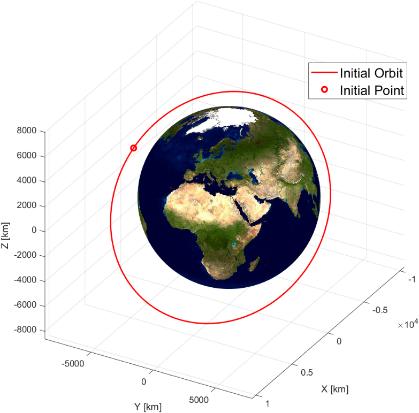
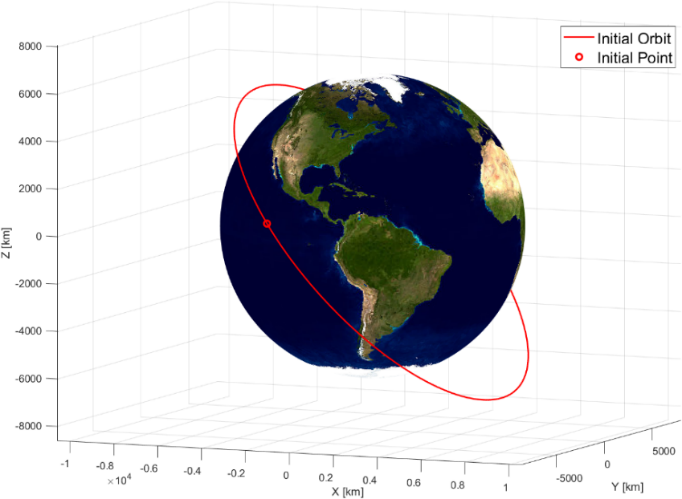


Figure 1: Initial Orbit Figure 2: Initial Orbit

# Final orbit characterization

## Final orbital parameters

The goal orbit, that is geocentric just like the starting one, is defined by its orbital parameters:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| 10860 | 0.2332 | 0.5284 | 3.0230 | 0.4299 | 0.3316 |

The final position and velocity are calculated from these parameters:

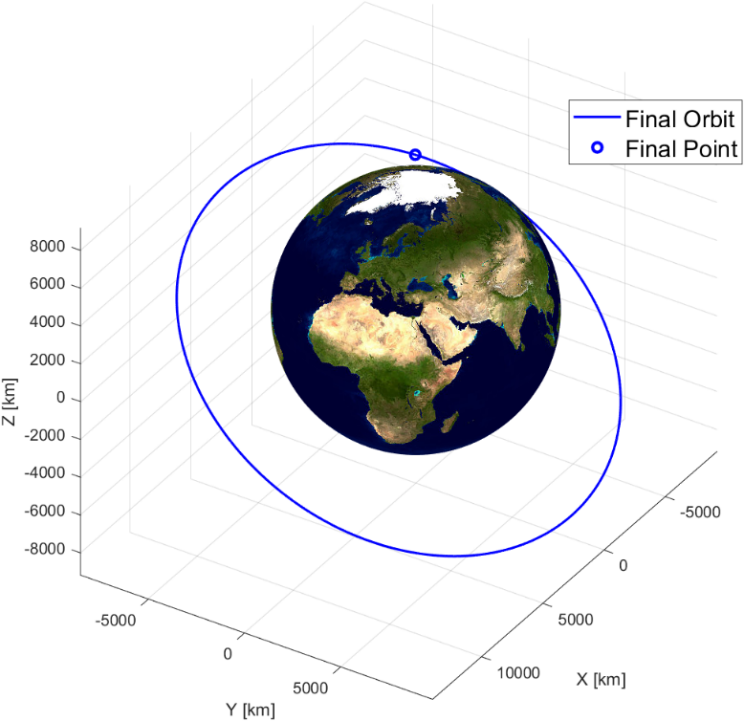
## 3.2 Data interpretation

The final geocentric orbit is elliptical, with an eccentricity value between 0 and 1 and a specific energy of:

It belongs to Medium Earth Orbit (MEO) category, as its apogee and its perigee are inside the range of 8000 – 42000 km:

According to the given value, it is nor a polar nor a geo-synchronous orbit and has a period of:

## 3.3 Graphical representation



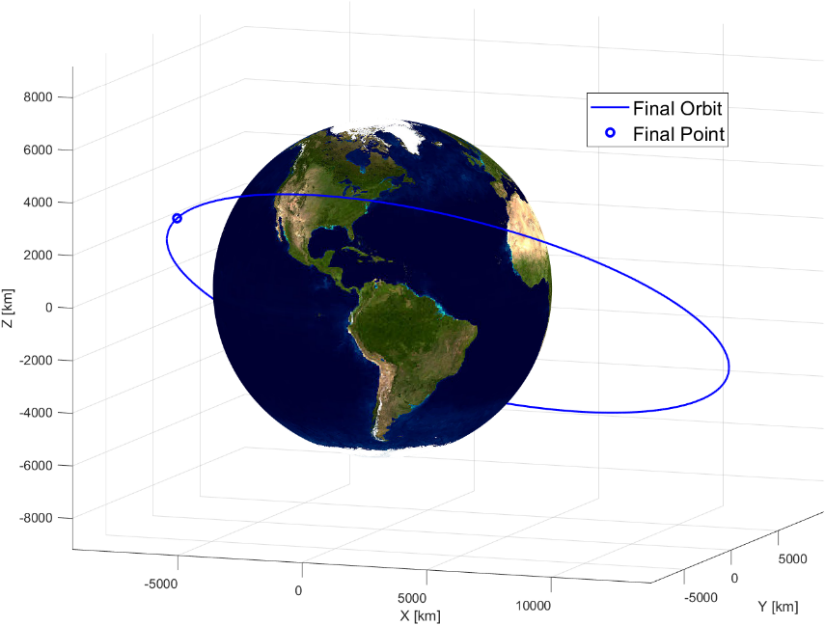


Figure 3: Final Orbit Figure 4: Final Orbit

# Transfer trajectory definition and analysis

## 4.1.1 Standard Strategy

It is possible to reach the final assigned point, located on the final orbit, from the initial point on the initial orbit, through a standard strategy using a specific permutation of the three known manoeuvres between orbits. The chosen standard strategy is composed of, in sequence, a bitangent transfer perigee to apogee, a change of the orbital plane and a change of the argument of periapsis. The data concerning it can be found in Table S.1

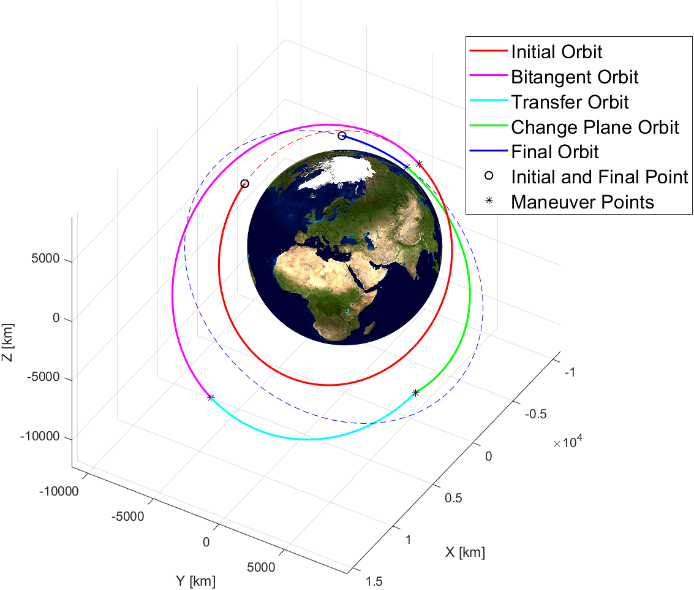
Each manoeuvre changes a specific set of orbital parameters.

1. Bitangent manoeuvre: to perform the first manoeuvre, it is needed to reach the first orbit’s perigee, due to the nature of the bitangent chosen manoeuvre, where the first impulse is made, moving the satellite on a new orbit, that differs from the previous orbit with a new major semiaxis and a new eccentricity. Once reached the apogee of the transfer orbit, through another impulse, the satellite is transferred to a third orbit with the same major semiaxis and same eccentricity as the final assigned orbit.
2. Change of orbital plane: given the finale inclination, it is needed to change the inclination of the orbit in a specific point. Through this manoeuvre the final inclination and final RAAN can be achieved.
3. Change of argument of periapsis: a final impulse is needed to reach the configuration of the final orbit, as the argument of the periapsis of the final orbit is different. The final point is then reached after a short travel on the final orbit.

## 4.1.2 Standard’s alternatives and decision explanations

Among the possible permutation it has been chosen to perform the strategy as described in paragraph 4.1.1. Data of this strategy are shown in table S.1. This strategy has been selected due to the lowest possible costs in term of change in velocity required, up to 27.3% lower than data reported in tables S.2 to S.8. It is possible to achieve this result thanks to some precautions, such as not making the change of inclination as first manoeuvre and done it in the farthest point possible, as in the strategies described in the tables S.1, S.4, S.5, S.6, with savings up to 13.6% on the of change inclination.

Moreover, the costs associated with the chosen bitangent manoeuvre, if in the perigee-apogee mode are lower than any other possible bitangent transfer, with a reduction in Δv of 2.15% compared to the apogee-perigee, and up to 34.7% for the others. There are no benefits in terms of Δv in doing a bitangent manoeuvre before the change of the orbital plane.

In regards of the time required by the strategy proposed in table S.1, these are higher than 21.1% compared to the other strategies in table S.9. The time required is greater because the orbits travelled are wider to reduce Δv. Costs associated with the change of argument of perigee are not the lowest (+47.6%) in table 1 but are necessaire as the Δv at the end of the process is lower.

Considering all the data in table S.1 to S.8, it is important to also note data of table S.2. This strategy requires, as reported in table S.9, the lowest time possible among the possible standards. Introducing as a merit parameter the product , that gives a mean value of the costs in terms of and , with reduction of 11.3% there is therefore a reduction of 11.3% compared to the parameter of the strategy considered above.

Figure 5: Standard Strategy 1

## Alternative Strategy 1

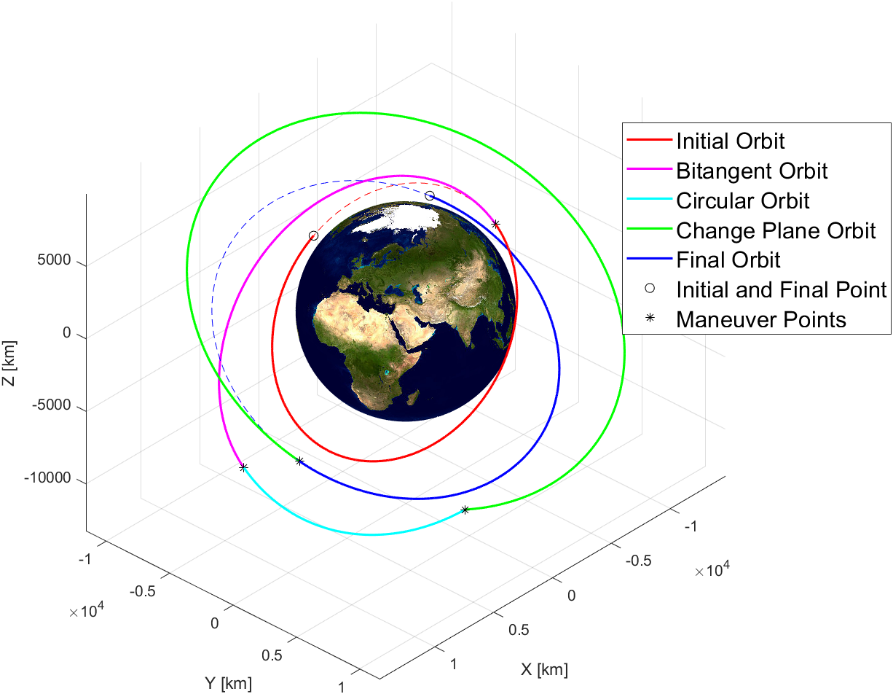
The first alternative strategy chosen is based on the use of a circular auxiliary orbit, that has the same radius as the apogee of the final orbit.

The reason for the choice is to avoid doing the manoeuvre of change of argument of the perigee, passing directly from the circular orbit changed of plan to the final orbit with all the final parameters.

The orbital transfer starts with a bitangent transfer from the perigee of initial orbit to the apogee of a circular transfer orbit, that has the apogee radius equal to the final orbit.

So done, a change of plane takes the circular orbit on the final plane.

Due to the initial inclination, it is far more convenient in terms of to perform the change in inclination in the first possible point, whereas the costs associated with do not change thanks to the circularity of the orbit.

After this manoeuvre, the value of is imposed as the final, this is done because the favourable conditions allowed by a circular orbit are lavished.

Once reached the apogee, after almost a full period on the transfer orbit, the last impulse is made to enter the final orbit and reach the point.

As could be seen in Figure 7, the orbits the satellite must travel are much wider than the ones of the proposed standard, resulting in an increment of time of 85%.

This strategy has a lower cost of change of plane than the standard of 6.7%, but other manoeuvres make this strategy more expensive.

Figure 6: Alternative Strategy 1

## Alternative Strategy 2: Secant Strategy

The second alternative strategy is a two-burn manoeuvre that has been chosen as the best compromise between the cost and the time of manoeuvre. In order to find the manoeuvre, it firstly has to be searched the two-burn manoeuvre that is able to minimize as much as possible the total cost (table A.5, figure 14). This manoeuvre has been realized through a MATLAB function that is able to return a set of possible secant manoeuvres (these ones discretize an infinite range of manoeuvres), given the initial point and the final point of the manoeuvre. Indeed, the burns can be arbitrarily directed into space: only the orbital plane remains constant, since it is the only one passing through the three known points (the initial and the final ones and the focus of the orbit). Therefore, the parameters remain unchanged, while the parameters will vary according to a chosen parameter.

So, the problem is underdetermined and therefore there are infinite orbits that can solve the problem: it is convenient to parametrize the argument of perigee by discretizing the range between 0 and , selecting successively the valid orbits. To do this, it has been used MATLAB to study the eccentricity as a function of through its graph (Figure 7); the shape of the latter remains similar for all the cases analysed, it always has just one range of for which the eccentricity is acceptable (between 0 and 1):

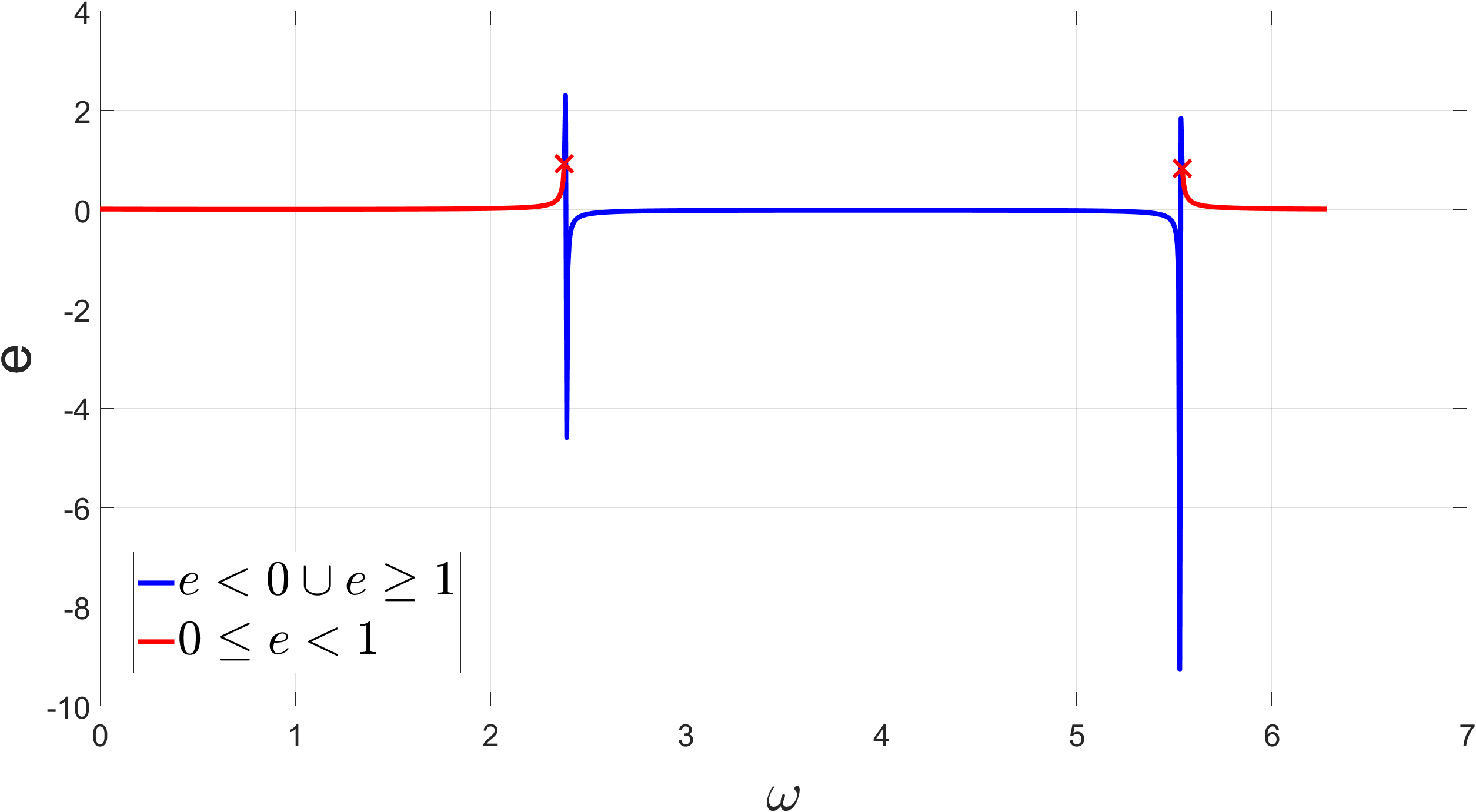
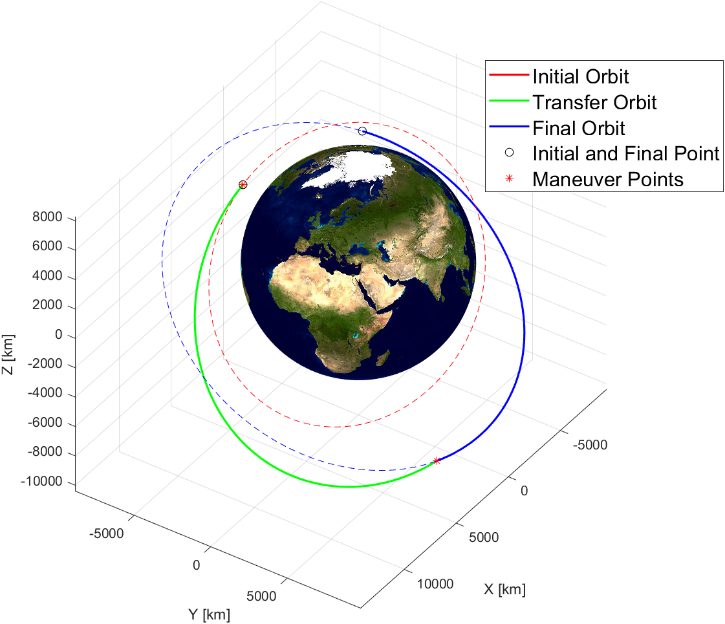


Figure 7: Graph of eccentricity as a function of

By isolating the range and discretizing it, it is possible to determine the remaining orbital parameters, to define a set of orbits passing through two points and to calculate the cost and the time of the various orbital transfers.

 By using the function described above, it has been defined an iterative process consisting of two nested for-loops that can vary the initial and the final points, discretizing the initial and the final orbits through their orbital parameters; among the analysed orbits, it has been found the one with the lowest total cost.

Starting from this orbit, it can be realized that the point of manoeuvre that has been chosen on the initial orbit is slightly rear from the initial point, and that the greatest amount of time used by the satellite is spent on the course the satellite accomplishes on the initial orbit (almost an entire orbital period). By knowing this, the initial point of manoeuvre has been fixed on the starting point, and the code has been re-adjusted by varying only the point on final orbit within the loop. The result is a secant transfer, whose total time is about halved (reduced by 46.96% compared to the previous one), while the total cost is increased by only 1.54%. Figure 8: Secant Strategy

## Alternative Strategy 3: Tangent Strategy

The last alternative strategy idea was to take advantage from the capability of a tangent manoeuvre to change all the orbital parameters (inclination ones excluded): therefore, the entire structure of this strategy has been projected to condense in a single manoeuvre the change of argument of perigee and the distancing from the main attractor (which is necessary to contain the cost of the subsequent orbital inclination change). The outcome of these first two manoeuvres will be an orbit in the same plane of the final orbit and, as previously planned, with the same argument of perigee that the final orbit has. In order to fix the semi-major axis and the eccentricity (the only parameters that differ between the current and the final orbit), a bitangent transfer will be performed from the apogee of the first orbit to the perigee of the second orbit.

The main difficulty in the design of this strategy is to obtain the desired change of argument of perigee during the tangent manoeuvre. It is easier to find the argument of perigee value needed in the plane of the initial orbit by proceeding backwards. By knowing the inclination and the RAAN of the two orbital planes and the argument of perigee of the final orbit, it is possible to obtain information about the initial argument of perigee and about the two manoeuvring angles:

Case with :

Since the transverse orbital speed is lower at (which is in the quadrant III), it has been selected to be the point where the orbital inclination change maneuver will be performed. After obtaining the information on the argument of perigee that should be reached in the initial orbital plane, it is necessary to design the tangent manoeuvre to achieve this value. Since the problem is under determined - and therefore infinite manoeuvres exist – it is chosen to parametrize the tangent burn ; a function has been defined in MATLAB to numerically solve the following system (simplified in an analytic way solving for ):

The result is a single nonlinear equation that can be studied and solved by using a numerical method similar to the one used on the eccentricity graph of the previous strategy: it always has two solutions, but only one can be considered acceptable (since the other one returns a negative eccentricity) or none (for too high values of the parameter ).

By choosing an acceptable initial burn value, the strategy is completely defined, and it is concluded after the change of orbital plane by a simple bitangent manoeuvre from apogee to perigee: therefore, the software MATLAB has been used to obtain the plot of the total cost of the strategy as a function of the tangent burn, and it is chosen the value by which such cost is minimized.

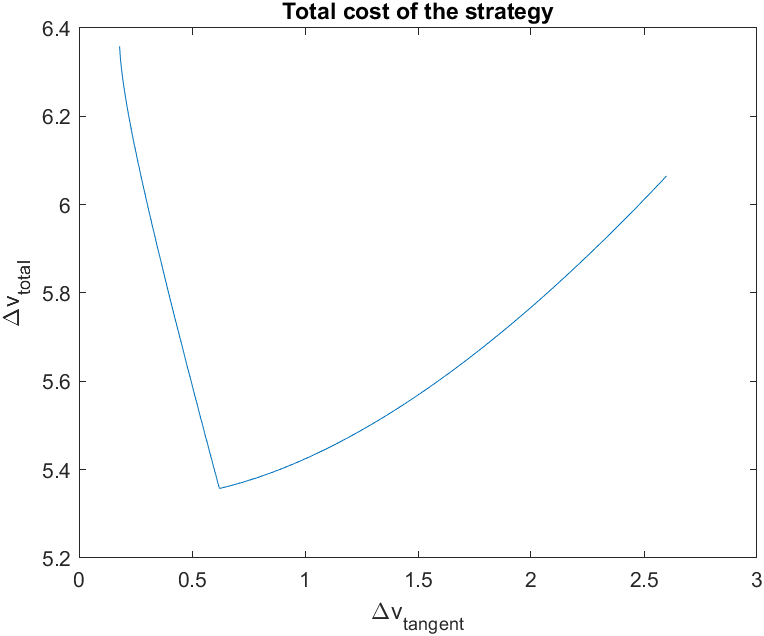
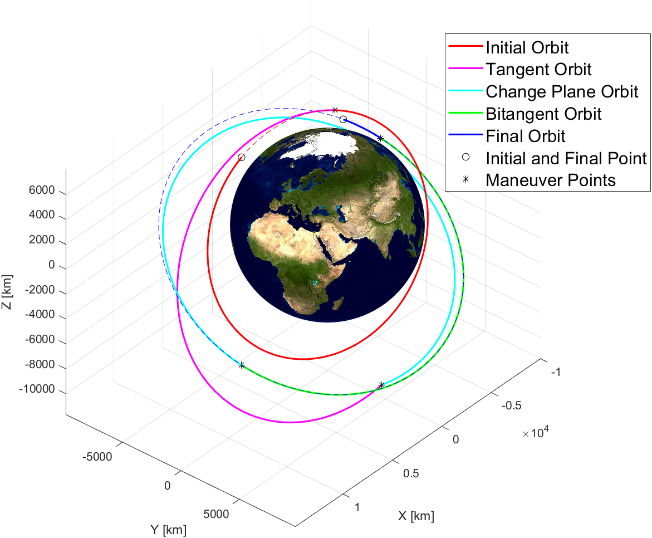


Figure 9: total cost of the strategy

From the data reported in the table A.3 it is also possible to observe that the second burn of the last manoeuvre is really small, because the two orbits are almost perfectly identical with a single-burn manoeuvre in the apogee: therefore, it can be deduced (the demonstration is not subject of this short relation) that the optimal strategy would be to fix the point of intersection between the plane-change orbit and the final one in their apogees, so as to adjust the semi-major axis and the eccentricity with a single burn. This constraint would make the strategy unique and fully defined by its equations.



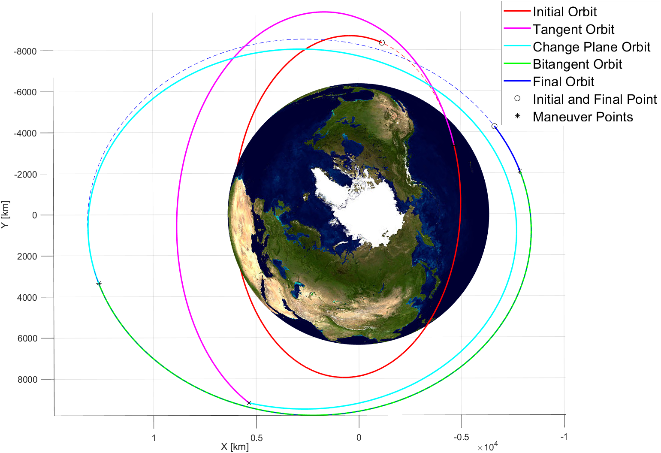


Figure 10: Tangent Strategy Figure 11: Tangent Strategy

# Conclusions

After comparing and choosing the best possible standard strategy and analyzing various alternative strategies, considerations can be made based on time and velocity costs reported below.

With an eye on the table in the appendix, for each strategy the costs associated with coplanar maneuvers are the lowest thanks to the similarities in shape and dimension of the two assigned orbits. In contrast, costs related to changing orbital plane are ruling, up to 78% of the total in the standard strategy, due to the difference in inclination of 18.3493° of the two orbital planes and the nearness of the main attractor.

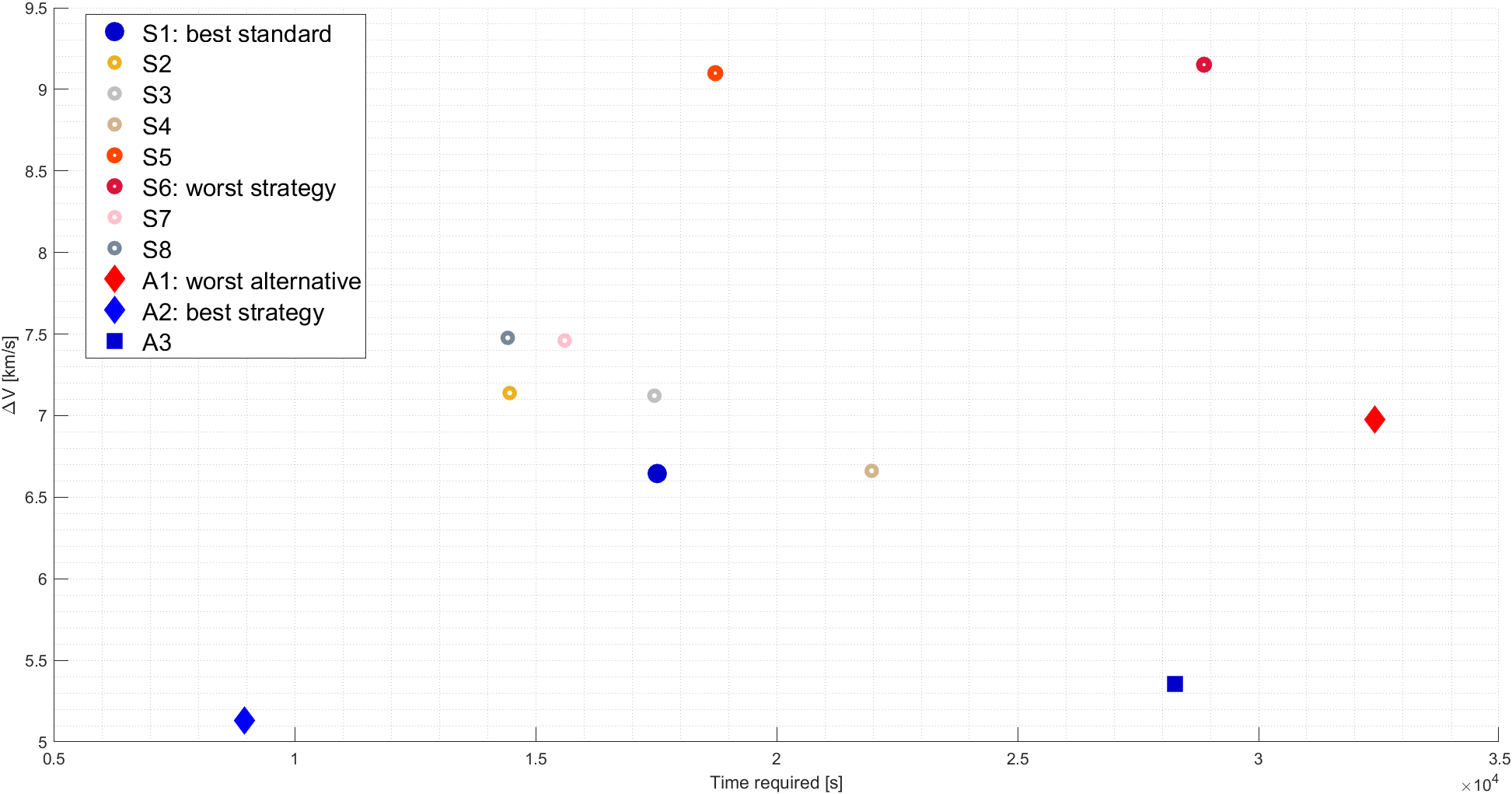
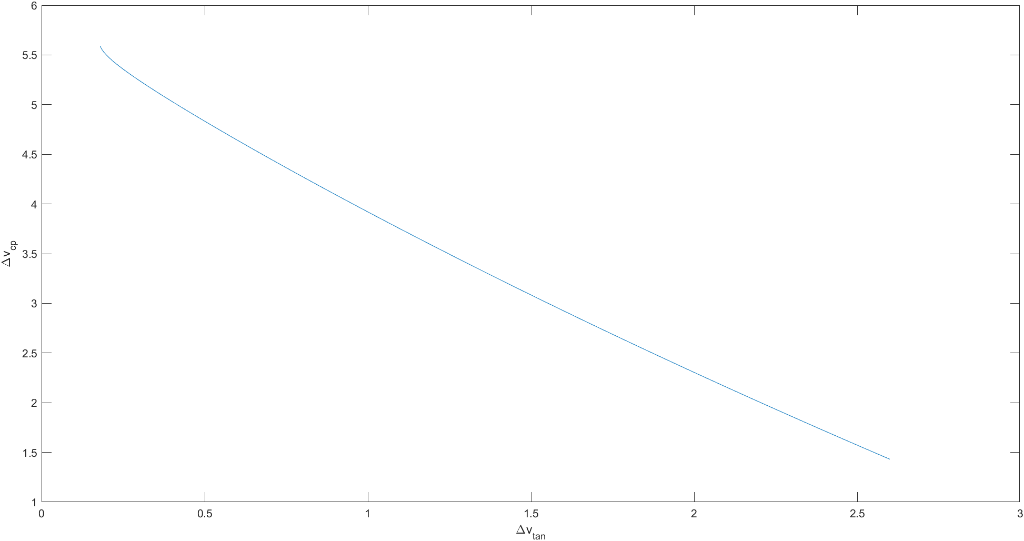


Figure 12: Comparison of strategies

As could be seen from the graphic above, the best strategy proposed is Alternative 2, the Secant, in which both time and velocity costs are reduced, thanks to the freedom given by the chosen method, where direction and modulus of the Δv vector can be decided, deleting constrains dictated by standard maneuvers, allowing an “ad hoc” strategy to minimize the total Δv.

Another viable option in terms of velocity costs is the Tangent Strategy, with a reduction in velocity costs of the 19.37% on the standard strategy: this is caused by the cheapness of the plane change, in which the burn is made farther from the main attractor and on an orbit with higher eccentricity. The bond of these two features allows a lower cross velocity: given an angle (angolo di perigee giusto?) and a change in inclination, increasing the eccentricity and the main semiaxis, the Δv of the maneuver decreases as could be seen in the graphic in Figure 13. Figure 13:

Alternative strategy 1 does not provide any benefit due to the shape, dimension and maneuvers made.

Both S1 and S4 are viable strategies as the Δv required are similar, however the position of the maneuvers negatively affects S4, making S1 the better option.

# Appendix

## Standard strategy tables

**S.1: Standard Strategy 1 (bitangent PA-change plane-change arg of perigee)**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| t (s) | a (km) | e (-) | i (rad) | Ω (rad) | ω (rad) | θ (rad) | Δv (km/s) |
| 0 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *1.8025* | - |
| 5697.7605 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *0* | 0.5863 |
| *10422.1787* | *0.2850* | *0.8487* | *1.5339* | *1.1849* | *0* |
| 10992.1880 | *10422.1787* | *0.2850* | *0.8487* | *1.5339* | *1.1849* | *3.1416* | 0.1642 |
| *10860* | *0.2332* | *0.8487* | *1.5339* | *1.1849* | *3.1416* |
| 14115.3731 | *10860* | *0.2332* | *0.8487* | *1.5339* | *1.1849* | *4.4179* | 5.1840 |
| *10860* | *0.2332* | *0.5284* | *3.0230* | *6.2190* | *4.4179* |
| 16892.4727 | *10860* | *0.2332* | *0.5284* | *3.0230* | *6.2190* | *0.2470* | 0.7105 |
| *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *6.0362* |
| 17523.1496 | *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *0.3316* | - |

**S.2: Standard Strategy 2 (change plane-change arg of perigee -bitangent AP)**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| t (s) | a (km) | e (-) | i (rad) | Ω (rad) | ω (rad) | θ (rad) | Δv (km/s) |
| 0 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *1.8025* | - |
| 3695.7504 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *4.4179* | 5.9993 |
| *8369.7488* | *0.1097* | *0.5284* | *3.0230* | *6.2190* | *4.4179* |
| 5937.1528 | *8369.7488* | *0.1097* | *0.5284* | *3.0230* | *6.2190* | *0.2470* | 0.3724 |
| *8369.7488* | *0.1097* | *0.5284* | *3.0230* | *0.4299* | *6.0362* |
| 9986.7633 | *8369.7488* | *0.1097* | *0.5284* | *3.0230* | *0.4299* | *3.1416* | 0.1886 |
| *8807.5701* | *0.0545* | *0.5284* | *3.0230* | *0.4299* | *3.1416* |
| 14099.8266 | *8807.5701* | *0.0545* | *0.5284* | *3.0230* | *0.4299* | *0* | 0.5784 |
| *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *0* |
| 14461.7429 | *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *0.3316* | - |

**S.3: Standard Strategy 3 (change plane-change arg of perigee -bitangent PA)**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| t (s) | a (km) | e (-) | i (rad) | Ω (rad) | ω (rad) | θ (rad) | Δv (km/s) |
| 0 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *1.8025* | - |
| 3695.7504 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *4.4179* | 5.9993 |
| *8369.7488* | *0.1097* | *0.5284* | *3.0230* | *6.2190* | *4.4179* |
| 5937.1528 | *8369.7488* | *0.1097* | *0.5284* | *3.0230* | *6.2190* | *0.2470* | 0.3724 |
| *8369.7488* | *0.1097* | *0.5284* | *3.0230* | *0.4299* | *6.0362* |
| 6176.5450 | *8369.7488* | *0.1097* | *0.5284* | *3.0230* | *0.4299* | *0* | 0.5863 |
| *10422.1787* | *0.2850* | *0.5284* | *3.0230* | *0.4299* | *0* |
| 11470.9723 | *10422.1787* | *0.2850* | *0.5284* | *3.0230* | *0.4299* | *3.1416* | 0.1642 |
| *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *3.1416* |
| 17464.4130 | *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *0.3316* | - |

**S.4: Standard Strategy 4 (bitangent AP-change plane-change arg of perigee)**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| t (s) | a (km) | e (-) | i (rad) |  | Ω (rad) | ω (rad) | θ (rad) | Δv (km/s) |
| 0 | *8369.7488* | *0.1097* | *0.8487* |  | *1.5339* | *1.1849* | *1.8025* | - |
| 5697.7605 | *8369.7488* | *0.1097* | *0.8487* |  | *1.5339* | *1.1849* | *3.1416* | 0.1886 |
| *8807.5701* | *0.0545* | *0.8487* |  | *1.5339* | *1.1849* | *3.1416* |
| 9810.8238 | *8807.5701* | *0.0545* | *0.8487* |  | *1.5339* | *1.1849* | *0* | 0.5784 |
| *10860* | *0.2332* | *0.8487* |  | *1.5339* | *1.1849* | *0* |
| 18565.5333 | *10860* | *0.2332* | *0.8487* |  | *1.5339* | *1.1849* | *4.4179* | 5.1840 |
| *10860* | *0.2332* | *0.5284* |  | *3.0230* | *6.2190* | *4.4179* |
| 21342.6330 | *10860* | *0.2332* | *0.5284* |  | *3.0230* | *6.2190* | *0.2470* | 0.7105 |
| *10860* | *0.2332* | *0.5284* |  | *3.0230* | *0.4299* | *6.0362* |
| 21973.3098 | *10860* | *0.2332* | *0.5284* |  | *3.0230* | *0.4299* | *0.3316* | - |

**S.5: Standard Strategy 5 (bitangent AA-change plane-change arg of perigee)**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| t (s) | a (km) | e (-) | i (rad) | Ω (rad) | ω (rad) | θ (rad) | Δv (km/s) |
| 0 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *1.8025* | - |
| 1887.5422 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *3.1416* | 0.9379 |
| *11340.1221* | *0.1809* | *0.8487* | *1.5339* | *4.3265* | *0* |
| 7896.6199 | *11340.1221* | *0.1809* | *0.8487* | *1.5339* | *4.3265* | *3.1416* | 0.1600 |
| *10860* | *0.2332* | *0.8487* | *1.5339* | *4.3265* | *3.1416* |
| 11019.8052 | *10860* | *0.2332* | *0.8487* | *1.5339* | *4.3265* | *4.4179* | 5.1840 |
| *10860* | *0.2332* | *0.5284* | *3.0230* | *3.0775* | *4.4179* |
| 15947.3993 | *10860* | *0.2332* | *0.5284* | *3.0230* | *3.0775* | *1.8178* | 2.8175 |
| *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *4.4654* |
| 18728.5707 | *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *0.3316* | - |

**S.6: Standard Strategy 6 (bitangent PP-change plane-change arg of perigee)**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| t (s) | a (km) | e (-) | i (rad) | Ω (rad) | ω (rad) | θ (rad) | Δv (km/s) |
| 0 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *1.8025* | - |
| 5697.7605 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *0* | 0.1904 |
| *7889.6266* | *0.0555* | *0.8487* | *1.5339* | *1.1849* | *0* |
| 9184.8726 | *7889.6266* | *0.0555* | *0.8487* | *1.5339* | *1.1849* | *3.1416* | 0.9592 |
| *10860* | *0.2332* | *0.8487* | *1.5339* | *4.3265* | *0* |
| 17939.5821 | *10860* | *0.2332* | *0.8487* | *1.5339* | *4.3265* | *4.4179* | 5.1840 |
| *10860* | *0.2332* | *0.5284* | *3.0230* | *3.0775* | *4.4179* |
| 18845.6580 | *10860* | *0.2332* | *0.5284* | *3.0230* | *3.0775* | *4.9594* | 2.8175 |
| *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *1.3238* |
| 28868.3598 | *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *0.3316* | - |

**S.7: Standard Strategy 7 (change plane- bitangent PA-change arg of perigee)**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| t (s) | a (km) | e (-) | i (rad) | Ω (rad) | ω (rad) | θ (rad) | Δv (km/s) |
| 0 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *1.8025* | - |
| 3695.7504 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *4.4179* | 5.9993 |
| *8369.7488* | *0.1097* | *0.5284* | *3.0230* | *6.2190* | *4.4179* |
| 5697.7605 | *8369.7488* | *0.1097* | *0.5284* | *3.0230* | *6.2190* | *0* | 0.5863 |
| *10422.1787* | *0.2850* | *0.5284* | *3.0230* | *6.2190* | *0* |
| 10992.1879 | *10422.1787* | *0.2850* | *0.5284* | *3.0230* | *6.2190* | *3.1416* | 0.1642 |
| *10860* | *0.2332* | *0.5284* | *3.0230* | *6.2190* | *3.1416* |
| 15041.7985 | *10860* | *0.2332* | *0.5284* | *3.0230* | *6.2190* | *0.2470* | 0.7105 |
| *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *6.0362* |
| 15603.0824 | *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *0.3316* | - |

**S.8: Standard Strategy 8 (change plane- bitangent AP-change arg of perigee)**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| t (s) | a (km) | e (-) | i (rad) | Ω (rad) | ω (rad) | θ (rad) | Δv (km/s) |
| 0 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *1.8025* | - |
| 3695.7504 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *4.4179* | 5.9993 |
| *8369.7488* | *0.1097* | *0.5284* | *3.0230* | *6.2190* | *4.4179* |
| 5697.7605 | *8369.7488* | *0.1097* | *0.5284* | *3.0230* | *6.2190* | *3.1416* | 0.1886 |
| *8807.5701* | *0.0545* | *0.5284* | *3.0230* | *6.2190* | *3.1416* |
| 10992.1879 | *8807.5701* | *0.0545* | *0.5284* | *3.0230* | *6.2190* | *0* | 0.5784 |
| *10860* | *0.2332* | *0.5284* | *3.0230* | *6.2190* | *0* |
| 15041.7985 | *10860* | *0.2332* | *0.5284* | *3.0230* | *6.2190* | *0.2470* | 0.7105 |
| *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *6.0362* |
| 15603.0824 | *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *0.3316* | - |

**S.9: Summary tables**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Strategy | Δt (s) | Δt (h) | Δv (km/s) | ΔvΔt (km) |
| S.1 | 17523.1496 | 4.8675 | 6.6450 | 116440 |
| S.2 | 14461.7429 | 4.0172 | 7.1386 | 103237 |
| S.3 | 17464.4130 | 4.8512 | 7.1222 | 124384 |
| S.4 | 21973.3098 | 6.1037 | 6.6614 | 146374 |
| S.5 | 18728.5707 | 5.2024 | 9.0993 | 170418 |
| S.6 | 28868.3598 | 8.0190 | 9.1511 | 264178 |
| S.7 | 15603.0824 | 4.3342 | 7.4603 | 116403 |
| S.8 | 14421.7183 | 4.0060 | 7.4767 | 107827 |

## Alternative strategy tables

**A.1: Alternative Strategy 1**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| t (s) | a (km) | e (-) | i (rad) | Ω (rad) | ω (rad) | θ (rad) | Δv (km/s) |
| 0 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *1.8025* | - |
| 5697.7605 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *0* | 0.5863 |
| *10422.1787* | *0.2850* | *0.8487* | *1.5339* | *1.1849* | *0* |
| 10992.1879 | *10422.1787* | *0.2850* | *0.8487* | *1.5339* | *1.1849* | *3.1416* | 0.8425 |
| *13392.5520* | *0* | *0.8487* | *1.5339* | *1.1849* | *3.1416* |
| 14125.2636 | *13392.5520* | *0* | *0.8487* | *1.5339* | *1.1849* | *4.4179* | 4.8691 |
| *13392.5520* | *0* | *0.5284* | *3.0230* | *6.2191* | *4.4179* |
| 26416.5152 | *13392.5520* | *0* | *0.5284* | *3.0230* | *6.2191* | *3.6356* | 0.6783 |
| *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *3.1416* |
| 32409.9559 | *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *0.3316* | - |

**A.2: Secant Strategy**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| t (s) | a (km) | e (-) | i (rad) | Ω (rad) | ω (rad) | θ (rad) | Δv (km/s) |
| 0 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *1.8025* | 0.8132 |
| *10269.8173* | *0.2189* | *0.7796* | *1.5492* | *2.1779* | *0.7990* |
| 4932.1735 | *10269.8173* | *0.2189* | *0.7796* | *1.5492* | *2.1779* | *3.3856* | 4.3174 |
| *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *3.8777* |
| 8964.9024 | *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *0.3316* | - |

**A.3: Tangent Strategy**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| t (s) | a (km) | e (-) | i (rad) | Ω (rad) | ω (rad) | θ (rad) | Δv (km/s) |
| 0 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *1.8025* | - |
| 6543.2337 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *0.8537* | 0.6212 |
| *10499.6909* | *0.2755* | *0.8487* | *1.5339* | *1.6789* | *0.3597* |
| 13660.9278 | *10499.6909* | *0.2755* | *0.8487* | *1.5339* | *1.6789* | *3.9239* | 4.6024 |
| *10499.6909* | *0.2755* | *0.5284* | *3.0230* | *0.4299* | *3.9239* |
| 22266.2293 | *10499.6909* | *0.2755* | *0.5284* | *3.0230* | *0.4299* | *3.1416* | 0.1338 |
| *10860.1616* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *3.1416* |
| 27897.8793 | *10860.1616* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *0* | 0.00004 |
| *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *0* |
| 28259.7957 | *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *0.3316* | - |

**A.4: Summary tables**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Strategy | Δt (s) | Δt (h) | Δv (km/s) | ΔvΔt (km) |
| Alternative 1 | 32409.9559 | 9.0027 | 6.9762 | 226097 |
| Alternativa 2 | 8964.9024 | 2.4878 | 5.1306 | 45952 |
| Alternativa 3 | 28259.7957 | 7.8499 | 5.3574 | 151400 |

**A.5:** **Secant strategy with minimized Δv**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| t (s) | a (km) | e (-) | i (rad) | Ω (rad) | ω (rad) | θ (rad) | Δv (km/s) |
| 0 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *1.8025* | - |
| 7091.8360 | *8369.7488* | *0.1097* | *0.8487* | *1.5339* | *1.1849* | *1.3599* | 0.7980 |
| *10299.9641* | *0.2391* | *0.7905* | *1.5902* | *1.9373* | *0.5691* |
| 12948.4314 | *10299.9641* | *0.2391* | *0.7905* | *1.5902* | *1.9373* | *3.6221* | 5.0530 |
| *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *3.9160* |
| 16889.7292 | *10860* | *0.2332* | *0.5284* | *3.0230* | *0.4299* | *0.3316* | - |

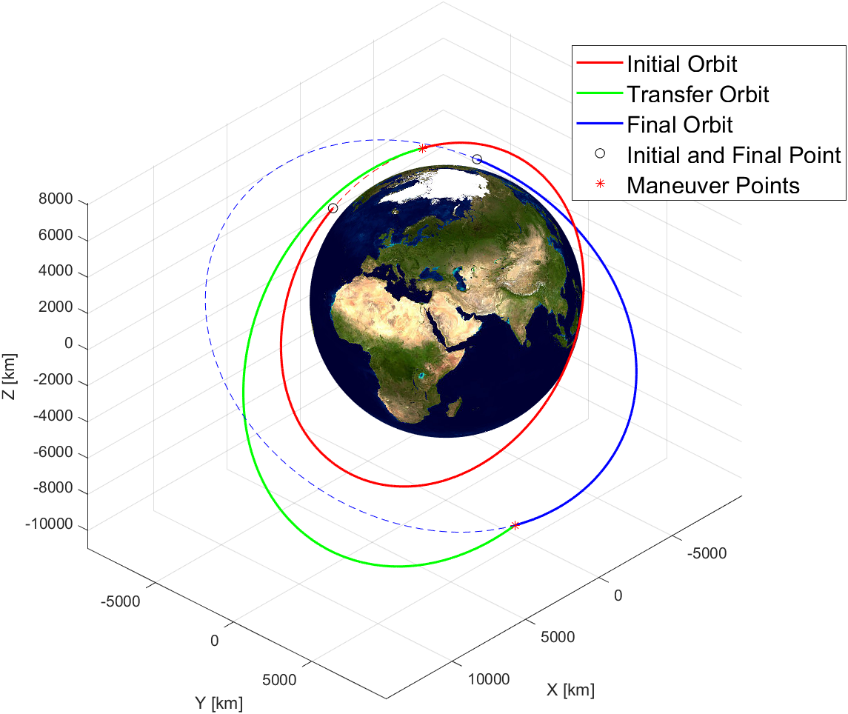
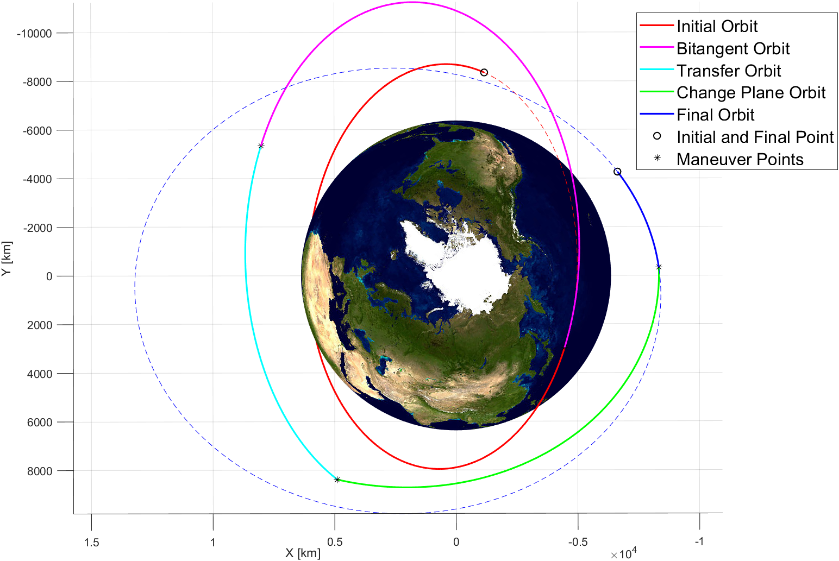


Figure 14:

Secant strategy with minimized Δv

 Figure 15:

Another point of view of the standard strategy S.1