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Dipartimento di Scienze e Tecnologie Aerospaziali  
Prova finale: Introduzione all’Analisi di Missioni Spaziali  
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Elaborato n. C13

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# Introduction

The project aims to study, optimize and choose various orbital transfer strategies, having as initial data a point on the initial orbit, which position and velocity vectors are given, and a point on the final orbit, which is defined by its orbital parameters.

First it will be analysed a strategy based on a set of standard manoeuvres.

Several other alternative strategies have been examined to try to optimize the two most significant parameters in their distinction: the manoeuvring cost (the total speed gap required to complete all the orbital changes) and the operating time (from the start point to the final point).

All calculations and plots are made using MATLAB software.

# Initial orbit characterization

## Initial orbital parameters

The assigned starting position and velocity vectors are the following:

It is possible to calculate the orbital parameters assigned to this specific couple of vectors:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| 8369.7448 | 0.1097 | 0.8487 | 1.5339 | 1.1849 | 1.8025 |

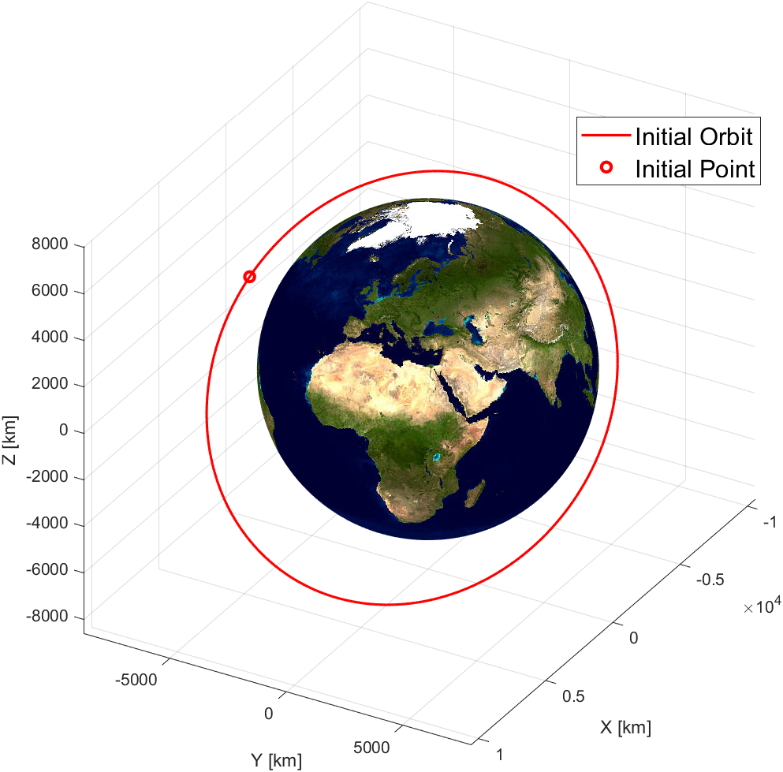
## Data interpretation

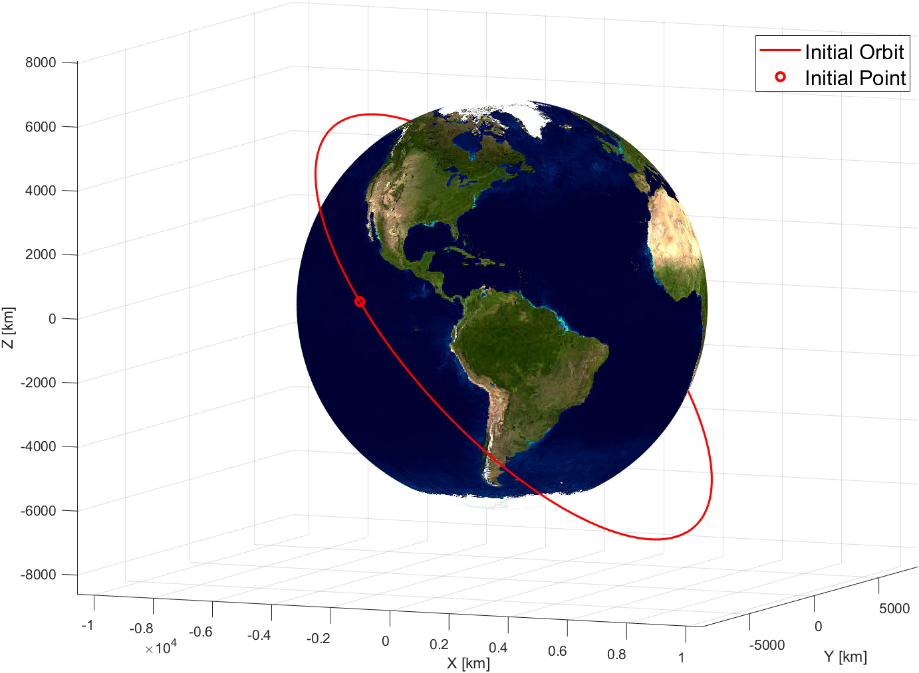
The starting geocentric orbit is elliptical, with an eccentricity value between 0 and 1 and a specific energy of:

It belongs to Medium Earth Orbit (MEO) category, as its apogee and its perigee are inside the range of 8000 – 42000 km:

According to the given value, it is nor a polar nor a geo-synchronous orbit and has a period of:

## Graphical representation





# Final orbit characterization

## Final orbital parameters

The goal orbit, that is geocentric just like the starting one, is defined by its orbital parameters:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| 10860 | 0.2332 | 0.5284 | 3.0230 | 0.4299 | 0.3316 |

The final position and velocity are calculated from these parameters:

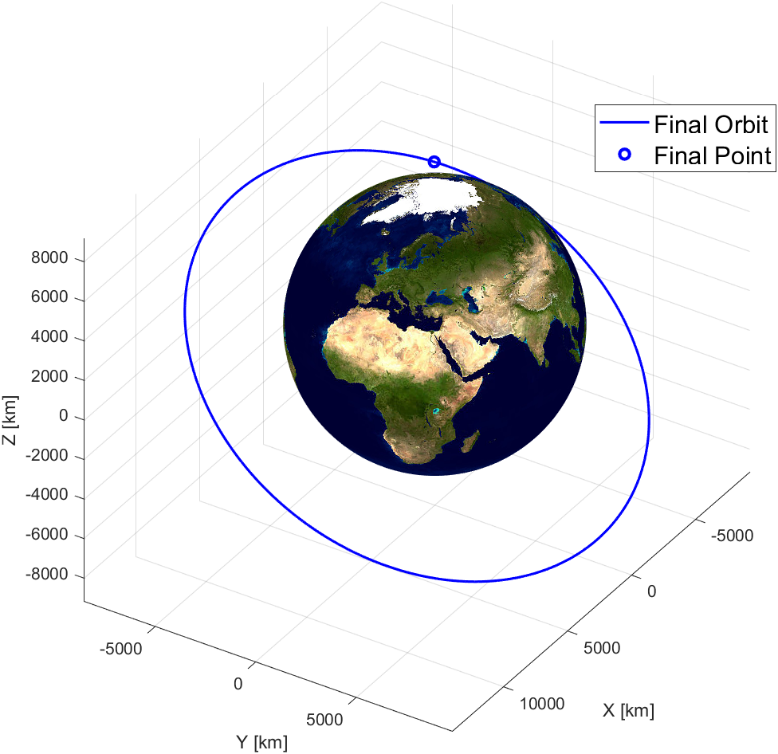
## 3.2 Data interpretation

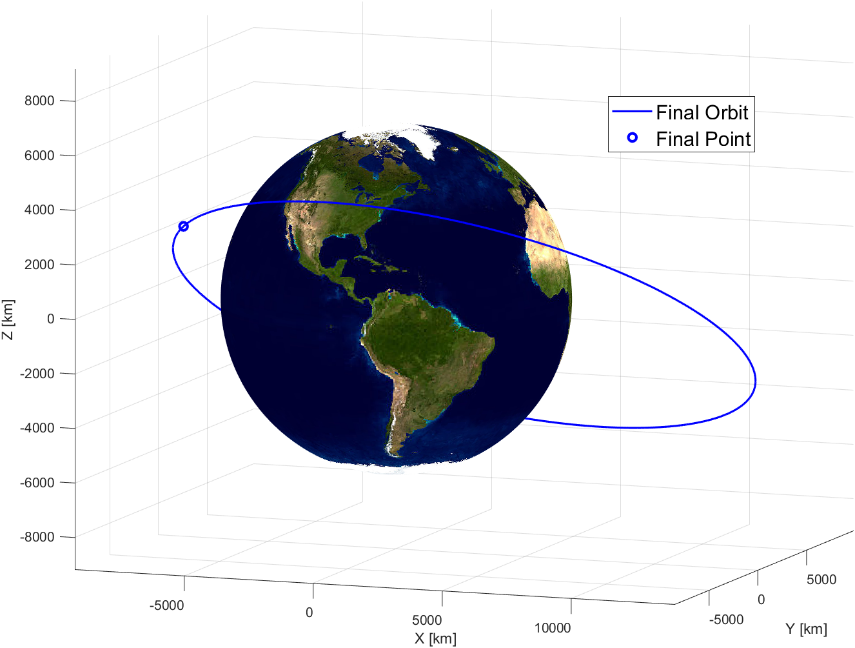
The final geocentric orbit is elliptical, with an eccentricity value between 0 and 1 and a specific energy of:

It belongs to Medium Earth Orbit (MEO) category, as its apogee and its perigee are inside the range of 8000 – 42000 km:

According to the given value, it is nor a polar nor a geo-synchronous orbit and has a period of:

## 3.3 Graphical representation





# Transfer trajectory definition and analysis

## 4.1.1 Standard Strategy

It is possible to reach the final assigned point, located on the final orbit, from the initial point on the initial orbit, through a standard strategy using a specific permutation of the three known manoeuvres between orbits. The chosen standard strategy is composed of, in sequence, a bitangent transfer perigee to apogee, a change of the orbital plane and a change of the pericentre’s anomaly. Each manoeuvre changes a specific set of orbital parameters.

1. Bitangent manoeuvre: to perform the first manoeuvre, it is needed to reach the first orbit’s pericentre, due to the nature of the bitangent chosen manoeuvre, where the first impulse is made, moving the satellite on a new orbit, that differs from the previous orbit with a new major semiaxis and a new eccentricity.
2. Transfer orbit: once reached the apogee of the second orbit, through another impulse, the satellite is transferred to a third orbit with the same major semiaxis and same eccentricity as the final assigned orbit.
3. Change in orbital plane: given the finale inclination, it is needed to change the inclination of the orbit in a specific point. Through this manoeuvre the final inclination and final RAAN can be achieved.
4. Change in argument of periapsis: a final impulse is needed to reach the configuration of the final orbit, as the argument of the periapsis of the final orbit is different. The final point is then reached after a short travel on the final orbit.

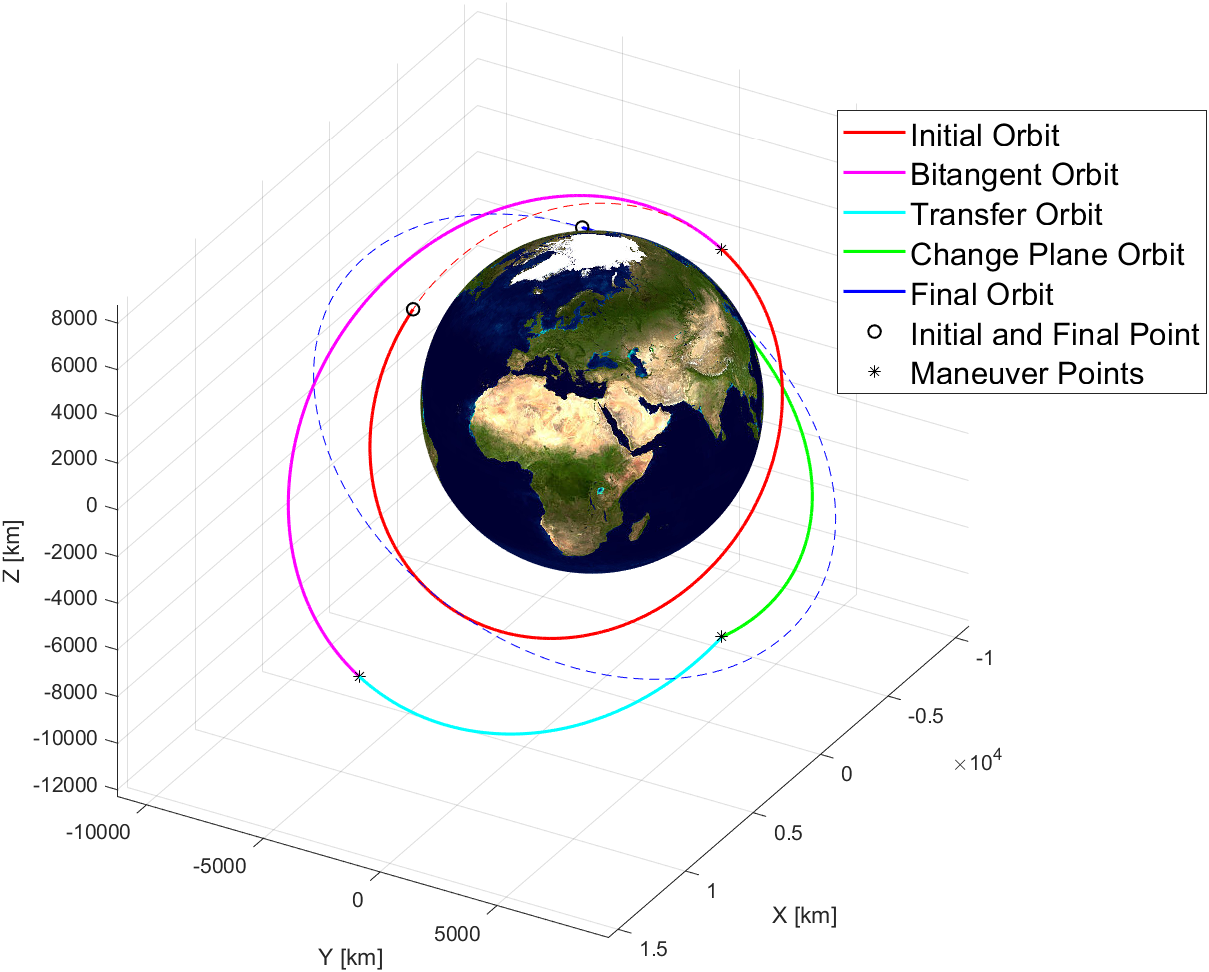
## 4.1.2 Standard’s alternatives and decision explanations

Among the possible permutation it has been chosen to perform the strategy as described in paragraph 4.1.1. Data of this strategy are shown in table 1. This strategy has been selected due to the lowest possible costs in term of change in velocity required, up to 27.3% compared with data reported below. It is possible to achieve this result thanks to some accouterments, such as the change of inclination, the most resource consuming transfer, done in the farthest point possible, thus not as the first manoeuvres, as in table 2, 3, 7 and 8, with savings up to 13.6% in . Moreover, the costs associated with the bitangent manoeuvre chosen, if done prior to the change in orbital plane, are significantly lower than any other bitangent transfer possible, with a reduction of up to 34.7%. There are no benefits in term of in doing a bitangent manoeuvre after the change in orbital plane.

In regards of the time required by the strategy proposed in table 1, these are higher than 21.1% compared to the strategy in table. The time required is greater because the orbits travelled are wider to reduce . Costs associated with the change in pericentre are not the lowest (+47.6%) in table 1 but are necessaire as the at the end of the process is lower.

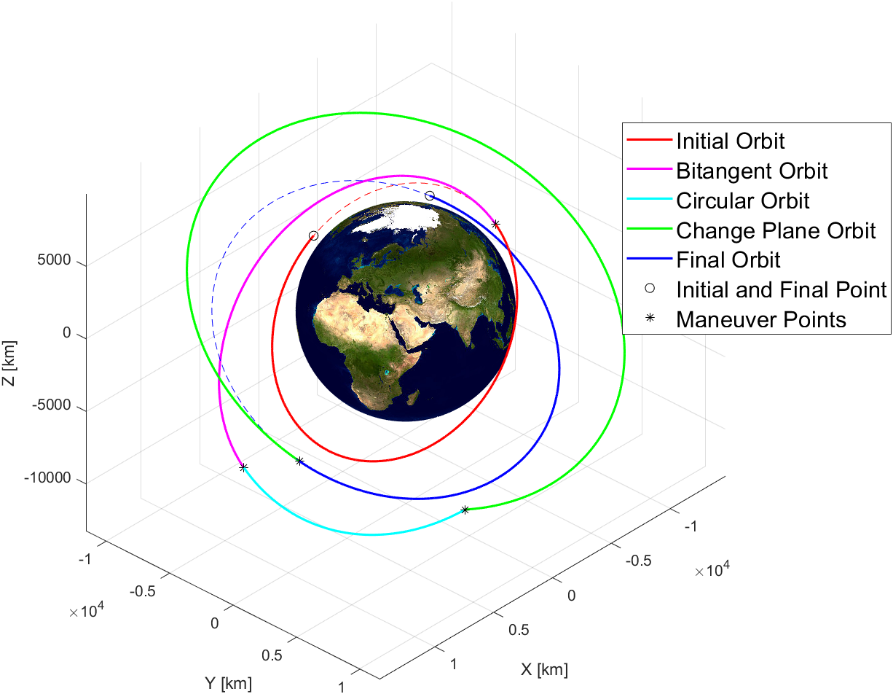
Considering all the data in table 1 to 6, it is important to note data of table 2. This strategy requires, as reported in table 7BANANA, the lowest time possible among the alternatives. Introducing as a merit parameter the product , that gives a mean value of the costs in terms of and , with reduction of 11.3%. Due to the higher costs in it is still not the recommended one.

## 4.1.3 Proposed strategy’s graphic



## Alternative Strategy 1

The first alternative strategy chosen is a little variation of the standard strategy above where a circularization is performed. The new circular orbit has the same radius as the apocentre of the goal orbit:

1. From the initial point it is reached the pericentre of the initial orbit, without any impulse. At the pericentre a bitangent pericentre to apocentre is made.
2. A second impulse is needed to reach the circular orbit.
3. Due to the initial inclination, it is far more convenient in terms of to perform the change in inclination in the first possible point. The costs associated with do not change thanks to the circularity of the orbit.
4. Once reached the apocentre, after almost a full period on the circular orbit, the last impulse is made to enter the goal orbit and reach the target.

As could be seen in the graph, the orbits the satellite must travel are much wider than the ones of the proposed standard, resulting in an increment of time of 85%.

This strategy has a lower cost in changing the orbital plane respect the standard manoeuvre by 6.7%, but almost every other cost is higher: the second impulse needed to reach the circular orbit denies any possible advantage with , with a final increment in costs of the 5% to the standard strategy.

## Alternative Strategy 2

## 4.4 Alternative Strategy 3

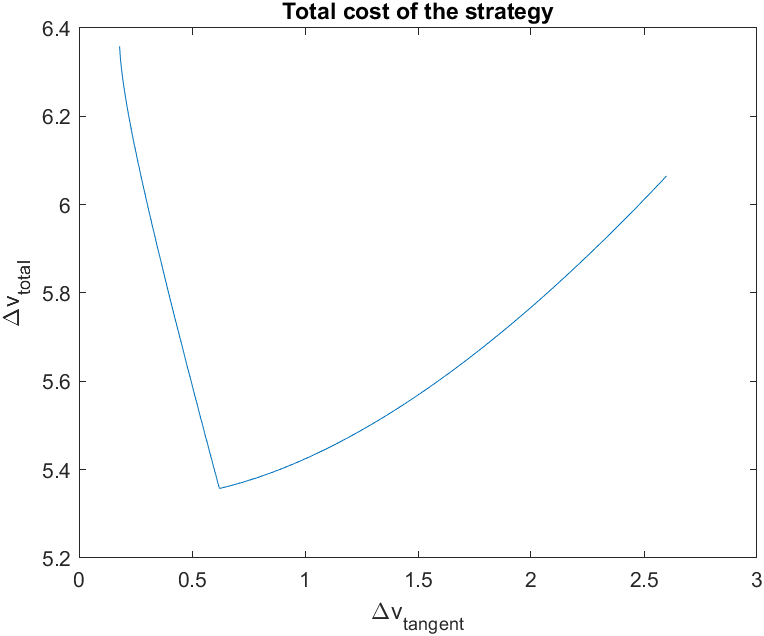
The second alternative strategy idea was to take advantage from the capability of a tangent maneuver to change all the orbital parameters (inclination ones excluded): therefore, the entire structure of this strategy has been projected to condense in a single maneuver the change of argument of perigee and the distancing from the main attractor (which is necessary to contain the cost of the subsequent orbital inclination change). The outcome of these first two maneuvers will be an orbit in the same plane of the final orbit and, as previously planned, with the same perigee argument that the final orbit has. In order to fix the semi-major axis and the eccentricity (the only parameters that differ between the current and the final orbit), a bitangent transfer will be performed from the apogee of the first orbit to the perigee of the second orbit.

The main difficulty in the design of this strategy is to obtain the desired change of perigee argument during the tangent maneuver. It is easier to find the perigee argument value needed in the plane of the initial orbit by proceeding backwards. By knowing the inclination and the RAAN of the two orbital planes and the perigee argument of the final orbit, it is possible to obtain information about the initial perigee argument and about the two maneuvering angles:

Case with :

Since the transverse orbital speed is lower at (which is in the quadrant III), it has been selected to be the point where the orbital inclination change maneuver will be performed. After obtaining the information on the perigee argument that should be reached in the initial orbital plane, it is necessary to design the tangent maneuver to achieve this value. Since the problem is under determined - and therefore infinite maneuvers exist – it is chosen to parametrize the tangent burn ; a function has been defined in MATLAB to numerically solve the following system (simplified in an analytic way solving for ):

The result is a single nonlinear equation that can be studied and solved by using a numerical method: it always has two solutions, but only one can be considered acceptable (since the other one returns a negative eccentricity) or none (for too high values of the parameter ).

By choosing an acceptable initial burn value, the strategy is completely defined, and it is concluded after the change of orbital plane by a simple bitangent maneuver from apogee to perigee: therefore, the software MATLAB has been used to obtain the plot of the total cost of the strategy as a function of the tangent burn, and it is chosen the value by which such cost is minimized.

From the data reported in the tables it is also possible to observe that the second burn of the last maneuver is really small, because the two orbits are almost perfectly identical with a single-pulse maneuver in the apocentre: therefore, it can be deduced (the demonstration is not subject of this short relation) that the optimal strategy would be to fix the point of intersection between the plane-change orbit and the final one in their apogees, so as to adjust the semi-major axis and the eccentricity with a single burn. This constraint would make the strategy unique and fully defined by its equations.

# 5. Conclusions

# 6. Appendix